

CANDIDATE
NAME

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MATHEMATICS

9709/31

Paper 3 Pure Mathematics 3 **(P3)**

May/June 2019

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO NOT WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 75.

This document consists of **18** printed pages and **2** blank pages.



- 1** Use the trapezium rule with 3 intervals to estimate the value of

$$\int_0^3 |2^x - 4| \, dx. \quad [3]$$

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 2 Showing all necessary working, solve the equation $\ln(2x - 3) = 2 \ln x - \ln(x - 1)$. Give your answer correct to 2 decimal places. [4]

[illegible]

- 3** Find the gradient of the curve $x^3 + 3xy^2 - y^3 = 1$ at the point with coordinates $(1, 3)$. [4]

[illegible]

- 4 By first expressing the equation $\cot \theta - \cot(\theta + 45^\circ) = 3$ as a quadratic equation in $\tan \theta$, solve the equation for $0^\circ < \theta < 180^\circ$. [6]

[illegible]

- 5 (i) Differentiate $\frac{1}{\sin^2 \theta}$ with respect to θ . [2]

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- (ii) The variables x and θ satisfy the differential equation

$$x \tan \theta \frac{dx}{d\theta} + \operatorname{cosec}^2 \theta = 0,$$

for $0 < \theta < \frac{1}{2}\pi$ and $x > 0$. It is given that $x = 4$ when $\theta = \frac{1}{6}\pi$. Solve the differential equation, obtaining an expression for x in terms of θ . [6]

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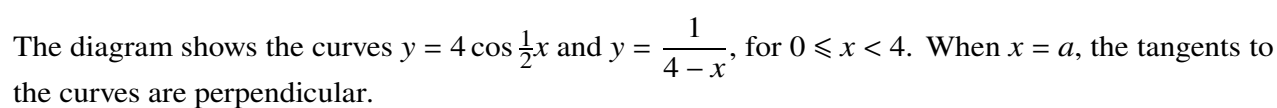
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6 (i) By first expanding $\sin(2x + x)$, show that $\sin 3x \equiv 3 \sin x - 4 \sin^3 x$. [4]

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This image shows a full page of a document template designed for handwriting practice or general note-taking. It consists of approximately 20 evenly spaced horizontal dotted lines across the entire width of the page. The background is plain white, and there are no margins, headers, footers, or other markings present.

[illegible]

- (ii) Verify by calculation that a lies between 2 and 3.

[2]

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- (iii) Use an iterative formula based on the equation in part (i) to determine a correct to 3 decimal places. Give the result of each iteration to 5 decimal places.

[3]

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8 Let $f(x) = \frac{16 - 17x}{(2 + x)(3 - x)^2}$.

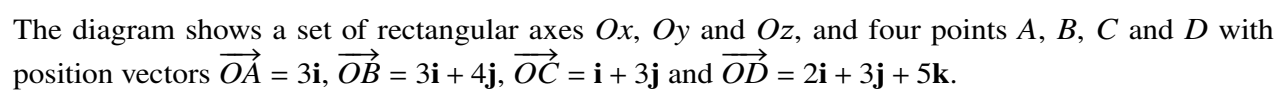
(i) Express $f(x)$ in partial fractions.

[5]

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- (ii) Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

[illegible]



- [illegible]

- (ii)** Calculate the acute angle between the planes BCD and $OABC$.

[4]

[illegible]

(i) Express u in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$, giving the exact values of r and θ . Hence or otherwise state the exact values of the modulus and argument of u^4 . [5]

[illegible]

- (ii) Verify that u is a root of the equation $z^3 - 8z + 8\sqrt{3} = 0$ and state the other complex root of this equation. [3]

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- (iii) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - u| \leq 2$ and $\text{Im } z \geq 2$, where $\text{Im } z$ denotes the imaginary part of z . [5]

[illegible]

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